$u_7 = \frac{5}{6}u_5$

.....

.....

.....

$u_{2n+1} = \frac{2n-1}{2n}u_{2n-1}$

En multipliant tous les termes puis en simplifiant on obtient

$u_{2n+1} = \frac{1}{2} \times \frac{3}{4} \times \frac{5}{6} \times \ldots\ldots \times \frac{2n-1}{2n}u_1$

$u_{2n+1} = \frac{1 \times \mathbf{2} \times 3 \times \mathbf{4} \times 5 \times \mathbf{6} + \ldots\ldots \times (\mathbf{2n-2}) \times (2n-1) \times \mathbf{2n}}{2 \times \mathbf{2} \times 4 \times \mathbf{4} \times 6 \times \mathbf{6} \times \ldots\ldots \times 2n \times \mathbf{2n}} \times \frac{\pi}{2}$

$u_{2n+1} = \frac{(2n)!}{2^{2n}(n!)^2} \times \frac{\pi}{2}.$

$u_4 = \frac{2}{3}u_2$

$u_6 = \frac{4}{5}u_4$

$u_8 = \frac{6}{7}u_6$

.....

.....

.....

$u_{2n+2} = \frac{2n}{2n+1}u_{2n}$

En multipliant tous les termes puis en simplifiant on obtient

$u_{2n+2} = \frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} \times \ldots\ldots \times \frac{2n}{2n+1} \times u_2$

$= \frac{2 \times \mathbf{2} \times 4 \times \mathbf{4} \times 6 \times \mathbf{6} \times \ldots\ldots \times 2n \times \mathbf{2n}}{\mathbf{2} \times 3 \times \mathbf{4} \times 5 \times \ldots\ldots \times (\mathbf{2n}) \times (2n+1)}$

$u_{2n+2} = \frac{2^{2n}(n!)^2}{(2n+1)!}.$

b- Soit $n \in IN$.

$\forall t \in IR$ on a : $0 < f(t) \leq 1$

$\Rightarrow \forall t \in IR$ on a : $0 < f^n(t)f(t) \leq 1 \times f^n(t)$

D'ou $\forall x \in IR^+$ on a $\int_0^x f^{n+1}(t)dt \leq \int_0^x f^n(t)dt$

$\Leftrightarrow \forall x \in IR^+$ on a $F_{n+1}(x) \leq F_n(x)$.

D'où $\lim\limits_{x \to +\infty} F_{n+1}(x) \leq \lim\limits_{x \to +\infty} F_n(x) \Leftrightarrow u_{n+1} \leq u_n$.

Par suite (u_n) est décroissante .

c- $\forall n \in IN$;

- $0 < u_{2n+2} \leq u_{2n+1}$ car (u_n) est décroissante .

$\Rightarrow \frac{u_{2n+2}}{u_{2n+1}} \leq 1$ car $u_{2n+1} > 0$

- $\frac{u_{2n+2}}{u_{2n+1}} = \frac{u_{2n+2}}{u_{2n}} \times \frac{u_{2n}}{u_{2n+1}} = \frac{2n}{2n+1} \times \frac{u_{2n}}{u_{2n+1}}$.